§6.3 FRICTION FACTORS FOR FLOW AROUND SPHERES

In this section we use the definition of the friction factor in Eq. 6.1-5 along with the dimensional analysis of §3.7 to determine the behavior of f for a stationary sphere in an infinite stream of fluid approaching with a uniform, steady velocity v_∞. We have already studied the flow around a sphere in §2.6 and §4.2 for Re < 0.1 (the "creeping flow" region). At Reynolds numbers above about 1 there is a significant unsteady eddy motion in the wake of the sphere. Therefore, it will be necessary to do a time average over a time interval long with respect to this eddy motion.

Recall from §2.6 that the total force acting in the z direction on the sphere can be written as the sum of a contribution from the normal stresses (F_n) and one from the tangential stresses (F_t). One part of the normal-stress contribution is the force that would be present even if the fluid were stationary, F_s. Thus the "kinetic force," associated with the fluid motion, is

$$F_k = (F_n - F_s) + F_t = F_{\text{form}} + F_{\text{friction}} \tag{6.3-1}$$

The forces associated with the form drag and the friction drag are then obtained from

$$F_{\text{form}}(t) = \int_0^{2\pi}\int_0^{\pi} (-\mathcal{P}|_{r=R} \cos\theta) R^2 \sin\theta \, d\theta \, d\phi \tag{6.3-2}$$

$$F_{\text{friction}}(t) = \int_0^{2\pi}\int_0^{\pi} \left(-\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]\Bigg|_{r=R} \sin\theta\right) R^2 \sin\theta \, d\theta \, d\phi \tag{6.3-3}$$

Since v_r is zero everywhere on the sphere surface, the term containing $\partial v_r/\partial\theta$ is zero.

If now we split f into two parts as follows

$$f = f_{\text{form}} + f_{\text{friction}} \tag{6.3-4}$$

then, from the definition in Eq. 6.1-5, we get

$$f_{\text{form}}(\breve{t}) = \frac{2}{\pi}\int_0^{2\pi}\int_0^{\pi} (-\breve{\mathcal{P}}|_{\breve{r}=1} \cos\theta) \sin\theta \, d\theta \, d\phi \tag{6.3-5}$$

$$f_{\text{friction}}(\breve{t}) = -\frac{4}{\pi}\frac{1}{\text{Re}}\int_0^{2\pi}\int_0^{\pi} \left[\breve{r}\frac{\partial}{\partial \breve{r}}\left(\frac{\breve{v}_\theta}{\breve{r}}\right)\right]\Bigg|_{\breve{r}=1} \sin^2\theta \, d\theta \, d\phi \tag{6.3-6}$$

The friction factor is expressed here in terms of dimensionless variables

$$\breve{\mathcal{P}} = \frac{\mathcal{P}}{\rho v_\infty^2} \qquad \breve{v}_\theta = \frac{v_\theta}{v_\infty} \qquad \breve{r} = \frac{r}{R} \qquad \breve{t} = \frac{v_\infty t}{R} \tag{6.3-7}$$

and a Reynolds number defined as

$$\text{Re} = \frac{D v_\infty \rho}{\mu} = \frac{2R v_\infty \rho}{\mu} \tag{6.3-8}$$

To evaluate $f(\breve{t})$ one would have to know $\breve{\mathcal{P}}$ and $\breve{v}_\theta$ as functions of $\breve{r}$, θ, ϕ, and $\breve{t}$.

We know that for incompressible flow these distributions can *in principle* be obtained from the solution of Eqs. 3.7-8 and 9 along with the boundary conditions

B.C. 1: at $\breve{r} = 1$, $\breve{v}_r = 0$ and $\breve{v}_\theta = 0$ (6.3-9)

B.C. 2: at $\breve{r} = \infty$, $\breve{v}_z = 1$ (6.3-10)

B.C. 3: at $\breve{r} = \infty$, $\breve{\mathcal{P}} = 0$ (6.3-11)

and some appropriate initial condition on $\breve{\mathbf{v}}$. Because no additional dimensionless groups enter via the boundary and initial conditions, we know that the dimensionless pressure and velocity profiles will have the following form:

$$\breve{\mathcal{P}} = \breve{\mathcal{P}}(\breve{r}, \theta, \phi, \breve{t}; \text{Re}) \qquad \breve{\mathbf{v}} = \breve{\mathbf{v}}(\breve{r}, \theta, \phi, \breve{t}; \text{Re}) \tag{6.3-12}$$

When these expressions are substituted into Eqs. 6.3-5 and 6, it is then evident that the friction factor in Eq. 6.3-4 must have the form $f(\breve{t}) = f(\text{Re}, \breve{t})$, which, when time averaged over the turbulent fluctuations, simplifies to

$$f = f(\text{Re}) \tag{6.3-13}$$

by using arguments similar to those in §6.2. Hence from the definition of the friction factor and the dimensionless form of the equations of change and the boundary conditions, we find that f must be a function of Re alone.

Many experimental measurements of the drag force on spheres are available, and when these are plotted in dimensionless form, Fig. 6.3-1 results. For this system there is no sharp transition from an unstable laminar flow curve to a stable turbulent flow curve as for long tubes at a Reynolds number of about 2100 (see Fig. 6.2-2). Instead, as the approach velocity increases, f varies smoothly and moderately up to Reynolds numbers of the order of 10^5. The kink in the curve at about $\text{Re} = 2 \times 10^5$ is associated with the shift of the boundary layer separation zone from in front of the equator to in back of the equator of the sphere.[1]

We have juxtaposed the discussions of tube flow and flow around a sphere to emphasize the fact that various flow systems behave quite differently. Several points of difference between the two systems are:

Flow in Tubes	Flow Around Spheres
• Rather well defined laminar–turbulent transition at about Re = 2100	• No well defined laminar–turbulent transition
• The only contribution to f is the friction drag (if the tubes are smooth)	• Contributions to f from both friction and form drag
• No boundary layer separation	• There is a kink in the f vs. Re curve associated with a shift in the separation zone

The general shape of the curves in Figs. 6.2-2 and 6.3-1 should be carefully remembered.

For the *creeping flow region*, we already know that the drag force is given by *Stokes' law*, which is a consequence of solving the continuity equation and the Navier–Stokes equation of motion without the $\rho D\mathbf{v}/Dt$ term. Stokes' law can be rearranged into the form of Eq. 6.1-5 to get

$$F_k = (\pi R^2)(\tfrac{1}{2}\rho v_\infty^2)\left(\frac{24}{Dv_\infty\rho/\mu}\right) \tag{6.3-14}$$

Hence for *creeping flow* around a sphere

$$f = \frac{24}{\text{Re}} \qquad \text{for Re} < 0.1 \tag{6.3-15}$$

and this is the straight-line asymptote as $\text{Re} \to 0$ of the friction factor curve in Fig. 6.3-1.

For higher values of the Reynolds number, Eq. 4.2-21 can describe f accurately up to about Re = 1. However, the empirical expression[2]

$$f = \left(\sqrt{\frac{24}{\text{Re}}} + 0.5407\right)^2 \qquad \text{for Re} < 6000 \tag{6.3-16}$$

[1] R. K. Adair, *The Physics of Baseball*, Harper and Row, New York (1990).

[2] F. F. Abraham, *Physics of Fluids*, **13**, 2194 (1970); M. Van Dyke, *Physics of Fluids*, **14**, 1038–1039 (1971).

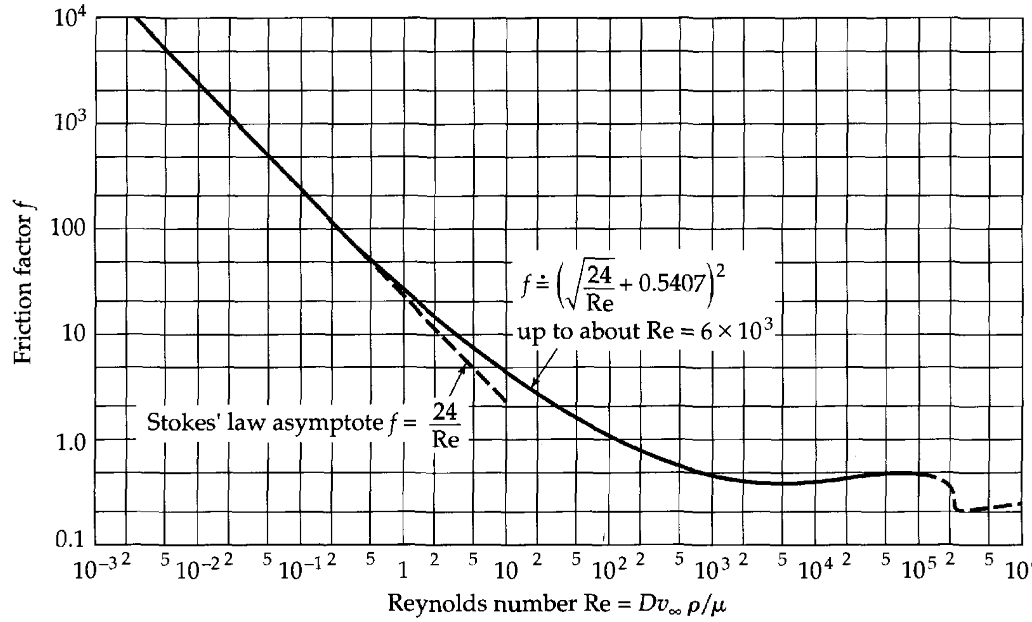

Fig. 6.3-1. Friction factor (or drag coefficient) for spheres moving relative to a fluid with a velocity v_∞. The definition of f is given in Eq. 6.1-5. [Curve taken from C. E. Lapple, "Dust and Mist Collection," in *Chemical Engineers' Handbook,* (J. H. Perry, ed.), McGraw-Hill, New York, 3rd edition (1950), p. 1018.]

is both simple and useful. It is important to remember that

$$f \approx 0.44 \qquad \text{for } 5 \times 10^2 < \text{Re} < 1 \times 10^5 \tag{6.3-17}$$

which covers a remarkable range of Reynolds numbers. Eq. 6.3-17 is sometimes called *Newton's resistance law;* it is handy for a seat-of-the-pants calculation. According to this, the drag force is proportional to the square of the approach velocity of the fluid.

Many extensions of Fig. 6.3-1 have been made, but a systematic study is beyond the scope of this text. Among the effects that have been investigated are wall effects[3] (see Prob. 6C.2), fall of droplets with internal circulation,[4] hindered settling (i.e., fall of clusters of particles[5] that interfere with one another), unsteady flow,[6] and the fall of nonspherical particles.[7]

EXAMPLE 6.3-1

Determination of the Diameter of a Falling Sphere

Glass spheres of density $\rho_{sph} = 2.62\ g/cm^3$ are to be allowed to fall through liquid CCl_4 at 20°C in an experiment for studying human reaction times in making time observations with stopwatches and more elaborate devices. At this temperature the relevant properties of CCl_4 are $\rho = 1.59\ g/cm^3$ and $\mu = 9.58$ millipoises. What diameter should the spheres be to have a terminal velocity of about 65 cm/s?

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[3] J. R. Strom and R. C. Kintner, *AIChE Journal*, **4**, 153–156 (1958).

[4] L. Landau and E. M. Lifshitz, *Fluid Mechanics*, Pergamon, Oxford, 2nd edition (1987), pp. 65–66; S. Hu and R. C. Kintner, *AIChE Journal*, **1**, 42–48 (1955).

[5] C. E. Lapple, *Fluid and Particle Mechanics*, University of Delaware Press, Newark, Del. (1951), Chapter 13; R. F. Probstein, *Physicochemical Hydrodynamics*, Wiley, New York, 2nd edition (1994), §5.4.

[6] R. R. Hughes and E. R. Gilliland, *Chem. Eng. Prog.*, **48**, 497–504 (1952); L. Landau and E. M. Lifshitz, *Fluid Mechanics*, Pergamon, Oxford, 2nd edition (1987), pp. 90–91.

[7] E. S. Pettyjohn and E. B. Christiansen, *Chem. Eng. Prog.*, **44**, 147 (1948); H. A. Becker, *Can. J. Chem. Eng.*, **37**, 885–891 (1959); S. Kim and S. J. Karrila, *Microhydrodynamics: Principles and Selected Applications*, Butterworth-Heinemann, Boston (1991), Chapter 5.